Lecture 10
Phys. 207: Waves and Light
Physics Department
Yarmouk University 21163 Irbid Jordan
Chapter 3: Waves II

3-1 Sound Waves
A sound wave is defined roughly as any longitudinal wave

Sound waves in modern life:
- Seismic prospecting teams use such waves to probe Earth’s crust for oil.
- Ships carry sound-ranging gear (sonar) to detect underwater obstacles.
- Submarines use sound waves to stalk other submarines.
- Explore the soft tissues of the human body.

Definitions
S: a point source that emits sound wave in all directions.

Wavefronts are surfaces over which the oscillations of the air due to the sound wave have the same value; such surfaces are represented by whole or partial circles in a two-dimensional drawing for a point source.

Rays are directed lines perpendicular to the wavefronts that indicate the direction of travel of the wavefronts.

Wavefronts - Rays
Fig. 1: A sound wave travels from a point source S through a 3D medium. The wavefronts form spheres centered on S; the rays are radial to S. The short, double-headed arrows indicate that elements of the medium oscillate parallel to the rays.
The speed of any mechanical wave, transverse or longitudinal, depends on two important properties of the medium:

- The inertial property of the medium (to store kinetic energy).
- The elastic property of the medium (to store potential energy).

In the case of a stretched medium, the speed of sound wave is given by:

\[ v = \sqrt{\frac{\tau}{\mu}} \]

\( \tau \) represents the elasticity property of the medium, while \( \mu \) represents the inertial property of the medium.

**Inertial and Elasticity Properties**

If the medium is air for example and the wave is longitudinal as for a sound wave then the inertial property is simply the volume density \( \rho \) of air.

The elasticity property here is the bulk modulus \( B \) defined by:

\[ B = -\frac{\Delta \rho}{\Delta V/V} \]

Where \( \Delta V/V \) is the fractional change in volume produced by a change in pressure \( \Delta \rho \).

**The Bulk Modulus**

Potential energy is associated with periodic compressions and expansions of small elements of the air. The property that determines the extent to which an element of a medium changes in volume when the pressure (force per unit area) on it changes is the bulk modulus \( B \).

\[ B = -\frac{\Delta \rho}{\Delta V/V} \]

The minus sign illustrates the fact that when we increase the pressure on air the volume decreases and vice-versa.

The speed of sound wave in any medium is given by:

\[ v = \sqrt{\frac{B}{\rho}} \]
The Speed of Sound

<table>
<thead>
<tr>
<th>Medium</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gases</strong></td>
<td></td>
</tr>
<tr>
<td>Air (0°C)</td>
<td>331</td>
</tr>
<tr>
<td>Air (20°C)</td>
<td>343</td>
</tr>
<tr>
<td>Helium</td>
<td>965</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>1284</td>
</tr>
<tr>
<td><strong>Liquids</strong></td>
<td></td>
</tr>
<tr>
<td>Water (0°C)</td>
<td>1402</td>
</tr>
<tr>
<td>Water (20°C)</td>
<td>1482</td>
</tr>
<tr>
<td>Seawater*</td>
<td>1522</td>
</tr>
<tr>
<td><strong>Solids</strong></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>6420</td>
</tr>
<tr>
<td>Steel</td>
<td>5941</td>
</tr>
<tr>
<td>Granite</td>
<td>6000</td>
</tr>
</tbody>
</table>

*At 0°C and 1 atm pressure, except where noted.

**TABLE 2-1 The Speed of Sound**

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Formal Derivation of \( v \)

The leading face of the slice enters the pulse (Figure b).

The average forces acting on the leading and the trailing faces are respectively:

\[
F_l = pA, \quad F_t = -(p + \Delta p)A
\]

The average net force on the system (the slice) is:

\[
\sum F_{rel} = pA - (p + \Delta p)A = -\Delta pA = \Delta m a
\]

\[
\Delta m a = pA \Delta x \Rightarrow \frac{a}{p} = \frac{\Delta p}{p \Delta x} = \frac{\Delta p}{\Delta x} \Rightarrow v = \sqrt{\frac{B}{\rho}}
\]
Displacement and Pressure Variation

\[ s(x, t) = s_m \cos(kx - \omega t) \]

Amplitude

\[ \Delta p(x, t) = \Delta p_m \sin(kx - \omega t) \]

Oscillating term

\[ \Delta p_m = (\rho \omega) s_m \]

Pressure variation

The pressure variation will be derived in the next section.

Pressure Variation

\[ \Delta p = -B \frac{\Delta V}{V} \]

\[ AV = A \Delta s, \quad V = A \Delta x \]

\[ \Delta p = -B \frac{\Delta s}{\Delta x} = -B \frac{\partial k}{\partial x} \]

\[ \frac{\partial k}{\partial x} = \frac{\partial}{\partial x} [s_m \cos(kx - \omega t)] = -k s_m \sin(kx - \omega t) \]

\[ \Delta p = B k s_m \sin(kx - \omega t) \]

\[ \Delta p_m = B k s_m = \left( \frac{\partial k}{\partial x} \right) s_m \]
The intensity $I$ of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface. We can write this as:

$$I = \frac{P}{A}$$

where $P$ is the average Power.

We will show that the intensity $I$ is related to the displacement amplitude $S_m$ by the relation:

$$I = \frac{1}{2} \rho v \omega^2 S_m^2$$

### Variation of Intensity with Distance

Fig. 18-10 A point source $S$ emits sound waves uniformly in all directions. The waves pass through an imaginary sphere of radius $r$ that is centered on $S$.

The intensity of sound at any point on the surface of a sphere of radius $r$ is given by:

$$I = \frac{P_i}{4\pi r^2}$$  \hspace{1cm} \text{Fig. 4}$$

### What kind of sounds human can hear?!!!!

We suggested earlier that the intensity of sound is given by:

$$I = f(v, f, S_m)$$

It is clear from the above equation that the intensity $I$ depends on the speed of sound wave $v$ and on the frequency $f$ of sound wave or the wave length $\lambda$, and the displacement amplitude $S_m$. 

$$I = \frac{1}{2} \rho v \omega^2 S_m^2$$
The Decibel Scale

The displacement amplitude which the human ear can hear ranges from about $10^{-5}$ m for the loudest tolerable sound to about $10^{-11}$ m for the faintest detectable sound, a ratio of $10^6$.

The ratio of the intensities at both limits is $10^{12}$. It is a huge range to consider.

We shall speak of the sound level $\beta$, instead of speaking of the intensities of sound wave at this large limit or ratio $10^{12}$.

Sound Level

The unit of sound level is dB. It is the abbreviation for decibel, a name that was chosen in recognition of the work of Alexander Graham Bell.

$I_0$ is a standard reference intensity $(=10^{-12} \text{ W/m}^2)$, chosen because it is near the lower limit of the human range of hearing.

For $I = I_0$, the above equation gives $\beta = 10 \log I = 0$

Some Sound Levels (dB)

<table>
<thead>
<tr>
<th>Sound Level</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hearing threshold</td>
<td>0</td>
</tr>
<tr>
<td>Rustle of leaves</td>
<td>10</td>
</tr>
<tr>
<td>Conversation</td>
<td>60</td>
</tr>
<tr>
<td>Rock concert</td>
<td>110</td>
</tr>
<tr>
<td>Pain threshold</td>
<td>120</td>
</tr>
<tr>
<td>Jet engine</td>
<td>130</td>
</tr>
</tbody>
</table>

Derivation of $I = \frac{1}{2} \rho v \omega^2 \psi_m^2$

\[
dK = \frac{1}{2} \frac{dm}{dt} \psi_m^2 \quad dm = \rho A dx
\]
\[
\psi_m = \hat{\omega}_m \sin(k \cdot x - \omega t)
\]
\[
dK = \frac{1}{2} \rho A \omega^2 \psi_m^2 \sin^2(k \cdot x - \omega t) \Rightarrow \frac{dK}{dt} = \frac{1}{2} \rho A \omega^2 \psi_m^2 \sin^2(k \cdot x - \omega t)
\]
\[
\left(\frac{dK}{dt}\right)_{\text{avg}} = \frac{1}{4} \rho A \omega^2 \psi_m^2 \sin^2(k \cdot x - \omega t)
\]
\[
I = \frac{\left(\frac{dE}{dt}\right)_{\text{avg}}}{A} = \frac{2 \left(\frac{dK}{dt}\right)_{\text{avg}}}{A} = \frac{1}{2} \rho v \omega^2 \psi_m^2
\]
A point source emits 30.0 W of sound isotropically. A small microphone intercepts the sound in an area of 0.750 cm², 200 m from the source. Calculate (a) the sound intensity there and (b) the power intercepted by the microphone.

Solution: \( P_{av} = 30 \text{ W} \)

(a) \( I \) at the mic. is:

\[
I = \frac{P_{av}}{4\pi r^2} = \frac{30 \text{ W}}{4\pi (200 \text{ m})^2} = 5.96 \times 10^{-5} \text{ W/m}^2
\]

(b) the power intercepted by the microphone.

\[
P(r) = I(r)A = 5.96 \times 10^{-5} \text{ W/m}^2 \times 0.75 \text{ cm}^2 = 4.47 \times 10^{-9} \text{ W}
\]

Sound level:

\[
\beta = 10\log\left(\frac{5.96 \times 10^{-5}}{10^{-12}}\right) = 52.9 \text{ dB}
\]

Standing Waves Patterns

(a) The simplest standing wave pattern of displacement for (longitudinal) sound waves in a pipe with both ends open has an antinode (A) across each end and a node (N) across the middle.*

(b) The corresponding standing wave pattern for (transverse) string waves.

*The longitudinal displacements represented by the double arrows are greatly exaggerated.
Standing wave patterns for string waves superimposed on pipes to represent standing sound wave patterns in the pipes.

**Pipe with both ends open**

With both ends of the pipe open, any harmonic can be set up in the pipe.

When both ends are open this leads to:

\[ \lambda_n = \frac{2L}{n} \Rightarrow f_n = \frac{n v}{2L} \]

A tube 1.20 m long is closed at one end. A stretched wire is placed near the open end. The wire is 0.330 m long and has a mass of 9.60 g. It is fixed at both ends and oscillates in its fundamental mode. By resonance, it sets the air column in the tube into oscillation at that column’s fundamental frequency. Find (a) that frequency and (b) the tension in the wire.

Solution:

For the tube:

\[ f_s = \frac{n v}{4L}, \quad n = 1, 3, 5, 7, \ldots \]

\[ f_t = \frac{n v}{4L} \quad \text{for } n = 1, 5, 9, 13, \ldots \]

\[ \text{For example: } 7, 13, 19, 25, \ldots \]

\[ f_t = \frac{341}{4 \times 1.2} = 71.5 \text{ Hz} \]

For the wire:

\[ f = \frac{v}{2L} \]

\[ = \frac{341}{2 \times 0.33} = 647 \text{ Hz} \]

\[ \mu_f = \frac{0.609 g \times 71.5 \text{ Hz}^2}{0.009 \text{ m}^2} = 33.3 \text{ N} \]
If we listen, a few minutes apart, to two sounds whose frequencies are, say, 552 and 564 Hz, most of us cannot tell one from the other.

However, if the sounds reach our ears simultaneously, what we hear is a sound whose frequency is 558 Hz, the average of the two combining frequencies. We also hear a striking variation in the intensity of this sound—it increases and decreases in slow, wavering beats that repeat at a frequency of 12 Hz, the difference between the two combining frequencies.

Let wave 1 be given by:

\[ s_1 = s_m \cos \omega_1 t \]

and wave 2 be given by:

\[ s_2 = s_m \cos \omega_2 t \]

The resultant wave \( s' \) is given by:

\[ s' = s_1 + s_2 = s_m \left( \cos \omega_1 t + \cos \omega_2 t \right) \]

Using the identity

\[ \cos A + \cos B = 2 \cos \frac{A-B}{2} \cos \frac{A+B}{2} \]

we have

\[ s' = 2s_m \cos \frac{1}{2} (\omega_1 - \omega_2) t \cos \frac{1}{2} (\omega_1 + \omega_2) t \]

If the two frequencies are nearly equal, i.e. if \( \omega_1 = \omega_2 = \omega \) then

\[ \omega_1 + \omega_2 = 2 \omega \quad , \quad \omega_1 - \omega_2 = \omega' \]

Note that \( \omega' \) is very small. We thus have:

\[ s' = 2s_m \cos \omega' t \]

Which is a “decaying” sine wave.

\[ t' = \frac{2 \pi}{\omega'} \]

\[ c \]

Time
Example 3 - Problem 43

A tuning fork of unknown frequency makes three beats per second with a standard fork of frequency 384 Hz. The beat frequency decreases when a small piece of wax is put on a prong of the first fork. What is the frequency of this fork?

Solution:

\[ \omega_{\text{beat}} = 2\omega' = \omega_1 - \omega_2 \]
\[ f_{\text{beat}} = f_1 - f_2 = f_1 - 384 = 3 \]
\[ f_1 = 387 \]

What is Doppler Effect?

Doppler Effect is a general phenomenon which occurs each time there is a relative motion between a source and a detector.

This is true for waves of all types. We shall discuss here the Doppler effect for mechanical (sound) waves.

Johann Christian Doppler

3-8 Doppler Effect

Source and detector in motion

Sound source S emits sound wave with frequency \( f \) what is the sound wave frequency \( f' \) heard by the detector D.

frequency \( f' \) depends on the relative motion of \( S \) and \( D \)

These motion-related frequency changes are examples of the Doppler effect.
1. When the detector moves relative to the air and the source is stationary relative to the air, the motion changes the frequency at which the detector intercepts wavefronts and thus the detected frequency of the sound wave.

2. When the source moves relative to the air and the detector is stationary relative to the air, the motion changes the wavelength of the sound wave and thus the detected frequency.

Stationary Source Moving detector

Fig. 18-19 A stationary source of sound S emits spherical wavefronts, shown one wavelength apart, that expand outward at speed $v$.

A sound detector $D$, represented by an ear, moves with velocity $v_D$ toward the source. The detector senses a higher frequency because of its motion.

The frequency $f'$ detected by $D$ is the rate at which $D$ intercepts wavefronts. Obviously $f' > f$.

Moving Source Stationary detector

Let us now consider the situation in which $D$ is moving opposite the wavefronts.

In time $t$, the wavefronts move to the right a distance $vt$ but here the detector $D$ moves to the left a distance $v_D$. 
Thus in time $t$ the distance moved by the wavefronts relative to $D$ is $vt + v_D t$. The number of wavelengths in this relative distance $vt + v_D t$ is the number of wavelengths intercepted by $D$ in time $t$, and this number is:

$$\frac{(vt + v_D t)}{\lambda} = \frac{v + v_D}{\lambda}$$

The rate at which $D$ intercepts wavelengths in this situation is the frequency $f'$ given by:

$$f' = \frac{v + v_D}{\lambda}$$

$\text{Summary: Stationary source moving detector}$

Detector moves towards to the source

Detector moves away from the source

Moving Source Stationary detector

Now let $S$ move at speed $v_S$ toward $D$, which is stationary with respect to the body of air. Consider two waves emitted by the source: $W_1$ and $W_2$. During the period $T$, the wavefront $W_1$ moves a distance $vt$ and the source moves a distance $v_S t$. At the end of $T$ wavefront $W_2$ is emitted.

In the direction of motion of $S$, the distance between the 2 wavefronts (which is the wavelength detected by $D$) is

$$\lambda' = vt - v_S t$$

The frequency $f'$ detected by the detector is:

$$f' = \frac{v}{\lambda'} = \frac{v}{vt - v_S t} = \frac{v}{v/f - v_S f} = \frac{v}{v - v_S f}$$

Moving Source Stationary detector

When both are stationary, the frequency $f = v/\lambda$

$$\lambda = \frac{Nvt}{N} = vt$$

$$\lambda' = \frac{N(v - v_S)}{N} = \frac{(v - v_S)t}{N}$$

$$f' = \frac{v}{\lambda'} = \frac{v}{vt - v_S t} = \frac{v}{v - v_S f} > f$$

$\text{Stationary Source Stationary detector}$

Similarly, if $D$ is moving away from the source, then the wavefronts move a distance $vt - v_D t$ relative to $D$ in time $t$ and we have:

$$f' = \frac{v - v_D}{v} f$$

Moving Source Stationary detector

Stationary Source Moving detector
Moving Source Stationary detector

When both are stationary, the frequency \( f = v/\lambda \)

\[
f' = \frac{v}{\lambda} = \frac{v}{v - v_s} f = \frac{v}{v - v_s} - v f > f
\]

General Doppler Effect Equation

We can summarize all the previous results in one single equation, namely:

Detector moves towards to the source:

\[
f' = \frac{v + v_s}{v} f
\]

Source moves away from the detector:

When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from the other, the sign on its speed must give a downward shift in frequency.

Lecture 13

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Waves II - Problems

Problem 6

The speed of sound in a certain metal is \( v \). One end of a long pipe of that metal of length \( L \) is struck a hard blow. A listener at the other end hears two sounds, one from the wave that travels along the pipe and the other from the wave that travels through the air. (a) If \( v \) is the speed of sound in air, what time interval \( t \) elapses between the arrivals of the two sounds? (b) Suppose that \( t = 1.00 \text{ s} \) and the metal is steel. Find the length \( L \).
Problem 6 - Solution

\[ T_s = L/V_s, \quad T_m = L/v \]

\[ \Delta T = T_s - T_m = L \left( \frac{1}{v} - \frac{1}{V_s} \right) \]

\[ \Delta t = 1 \text{ s}, \quad V_{ Steel} = 5941 \text{ m/s}, \quad v = 343 \text{ m/s} \]

\[ L = \Delta t \frac{V}{v} = 1 \times \frac{5941 \times 343}{5941 - 343} = 364 \text{ m} \]

Problem 11

The pressure in a traveling sound wave is given by the equation

\[ \Delta P = (1.50 \text{ Pa}) \sin \left( (0.900 \text{ m}^{-1}) x - (315 \text{ s}^{-1}) t \right) \]

Find (a) the pressure amplitude, (b) the frequency, (c) the wavelength, and (d) the speed of the wave.

a) \( \Delta P_{\text{max}} = 1.5 \text{ Pa} \)

b) \( f = \frac{\omega}{2\pi} = \frac{315 \pi}{2\pi} = 157.5 \text{ Hz} \)

\( \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.9\pi} = 2.22 \text{ m} \)

c) \( v = \frac{\omega}{k} = \frac{315\pi}{0.9\pi} = 350 \text{ m/s} \)

Problem 28

See Example 1

Problem 38

See Example 2

Problem 43

See Example 4

Problem 51 – Doppler Effect

A French submarine and a U.S. submarine move toward each other during maneuvers in motionless water in the North Atlantic (Fig. 18-37). The French sub moves at 50.0 km/h, and the U.S. sub at 70.0 km/h. The French sub sends out a sonar signal (sound wave in water) at 1000 Hz. Sonar waves travel at 5470 km/h.
**Problem 51 – Solution**

(a) What is the signal's frequency as detected by the U.S. sub?

Solution:

\[ f_{US} = \frac{f_s \pm v_D}{v_s} = \frac{1000 \pm 54700}{5470} = 5540 \text{ Hz} = 1022 \text{ Hz} \]

(b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?

\[ f_{ref} = f_{US} = \frac{54700 \pm 5520}{5470} = 1022 \text{ Hz} \]

**Problem 54 – Doppler Effect**

A bat is flitting about in a cave, navigating via ultrasonic bleeps. Assume that the sound emission frequency of the bat is 39000 Hz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.025 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

\[ f_r = f_b \frac{v \left( v + v_w \right)}{v - v_b} = f_b \frac{v}{v - v_b} = 39000 \times \frac{343}{343 - 343 \times 0.025} = 40000 \text{ Hz} \]

\[ f_{bat \text{ ref}} = f_b \frac{v \left( v + v_w \right)}{v - v_b} = 40000 \times \frac{343 + 343 \times 0.025}{343} = 41000 \text{ Hz} \]