Overview

Introduction

- Simplified Detector Model
- Interaction of Fast Electrons
- Pulse Height Spectra
- Counting Curves and Plateaus
- Energy Resolution
- Detection Efficiency
- Dead Time

Common Properties

Detectors have many common properties.
For all detectors we shall need to know the
* efficiency
* energy resolution

But also:
* modes of operation and
* methods of recording data.

Signal treatment is finally common to all types of detectors. But this is another course...
I. Simplified Detector Model

Schematic

Radiation hits the detector. Stopping time varies between a few ns to a few ps (See scheme). Times are practically so short that we can consider the deposition almost instantaneous!

First we consider that only one single particle (or one photon or quantum of radiation) is incident on the detector. The result of the interaction between the radiation and the material (of the detector) is a charge $Q$. An electric field is applied in order to "collect" the resulting electrons (currents).

Collection Time

The collection time from $t=0$ (creation of the charge) varies from a few ms for ionization chambers to ns for semiconductor diode detectors. This collection time is a function of 1) the mobility of charge carriers within the detector and 2) the average distance that the charge must travel before arriving at the collection electrodes! The figure below shows the flow of a current for a time equal to the charge collection time ($t_c$).

Real Situations

For any real situation, a flux of radiation is incident on the detector. If the rate of interaction is high then current is flowing in the detector from more than one interaction at a given time. For simplicity we consider that this rate is low enough such as the current resulting from each individual interaction is distinguishable from the others.

The figure below shows the flow of a current for a time equal to the charge collection time ($t_c$).
II. Modes of Detector Operation

- Pulse mode
- Current mode
- MSV (Mean Square Voltage) mode

General Features

- Current mode operation is used with many detectors when event rates are high.
- Detectors that are applied to radiation dosimetry are also normally operated in current mode.
- MSV mode is appropriate for enhancing the relative response of detector to large amplitude events. This mode is especially used in reactor instrumentation.
- Pulse mode is used when one wants to preserve the amplitude and timing of individual events.

Pulse Mode

Here the measurement instrumentation is designed in order to record each individual quantum of radiation that interacts in the detector. The total charge, \( Q \), related to the energy deposited in the detector, is recorded. Measurements of individual radiation quanta (radiation spectroscopy) must be done using detectors operated in pulse mode. The object is to measure the energy distribution of the incident radiation. Radiation spectroscopy, name reserved to such applications, constitutes the major part of this course.

We stress again the fact that this mode is practical and efficient in the case of low rate of interactions.

Current Mode

The sketch below shows the setup of a detector in current mode. A current measuring device (a picoammeter) is connected across the output terminals of a radiation detector.

If the measuring device has a fixed response time \( T \) then the recorded signal from a sequence of events will be a time-dependent current given by:

\[
I(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} i(t')dt'
\]
Averaging!

The response time $T$ being long compared to the average time between individual current pulses from the detector, the effect is to average out many of the fluctuations in the intervals between individual radiation interactions and to record an average current that depends on the product of the interaction rate and the charge per interaction.

Average Current

The average current is given by:

$$I_a = r Q = r \frac{E}{W}$$

(2)

$r =$ event rate
$Q = E q/W =$ charge produced for each event
$E =$ Average energy deposited per event
$W =$ average energy required to produce a unit charge pair (positive ion + $e^-$)
$e =$ fundamental charge = $1.6\times10^{-19}$ C

For a steady-state irradiation of the detector, the average current can also be rewritten as the sum of a constant current $I_0$ plus a time-dependent fluctuating component $\sigma_i(t)$ as in the sketch.

Random Component

$\sigma_i(t)$ is the random time-dependent variable that occurs as a consequence of the random nature of the radiation events interacting within the detector.

A statistical measure of this random component is the variance or the mean square value defined by:

$$\sigma_i^2(t) = \frac{1}{T} \int_{t-T}^{t} (\sigma_i(t'))^2 dt'$$

(3)

The standard deviation is thus:

$$\sigma_i(t) = \sqrt{\sigma_i^2(t)}$$

(4)
\( \sigma \), Rate and Time

The statistics being of Poisson type, the standard deviation is:

\[ \sigma_r = \sqrt{n} \]  \hspace{1cm} (5)

For a number of events occurring at rate \( r \) in an effective measurement time \( T \), the standard deviation is simply:

\[ \sigma_r = \sqrt{rT} \]  \hspace{1cm} (6)

In a hypothetical (and reasonable) approximation where we can consider that each recorded pulse contributes the same charge, the fractional standard deviation in the measured signal due to random fluctuations in pulse arrival time is given by:

\[ \frac{\sigma_f}{I_n} = \frac{1}{\sqrt{rT}} \]  \hspace{1cm} (7)

### B. Mean Square Voltage Mode

Suppose that we send the current signal through a circuit element that blocks the average current \( I_0 \) and only passes the fluctuating component \( \sigma_i(t) \).

The figure below schematizes the processing steps:

\[
\text{Ion Chamber} \xrightarrow{\text{Squaring Circuit}} \xrightarrow{\text{Averaging}} \text{Output}
\]

The results correspond to the quantity defined in Eq. 3

MSV: Signal Amplitude \( \propto Q^2 \)

Combining Eq. 2 and Eq. 7 we predict the magnitude of the signal derived in this way to be:

\[ \sigma_i^2(t) = \frac{rQ^2}{T} \]  \hspace{1cm} (8)

This mean square signal is directly proportional to the event rate \( r \) and more significantly proportional to the square of the charge \( Q \) produced in each event.

This is the MSV node, also called Campbelling mode (Campbell was the first to devise this model)

MSV: Application

The MSV mode is most useful when making measurements in mixed radiation environment when the charge produced by one type of radiation is much different from the second type.

A standard application is in the detection of neutron in reactors where one can distinguish easily the fast electrons from the smaller-amplitude gamma-rays events.
C. Pulse Mode

This mode is used when one wants to preserve information on the amplitude and timing of individual events. This is the mode which will be dealt with in this chapter.

The nature of the signal pulse from a single event depends on the input characteristics of the circuit to which the detector is connected (usually a preamplifier).

Typical Circuit in Pulse Mode

The following figure is a scheme of a circuit used in pulse mode.

\[ V(t) \] 

\[ R \] represents the input resistance of the circuit and \( C \) represents the equivalent capacitance of both the detector and the measuring circuit (including cables).

The potential difference \( V(t) \) across the load resistance is the fundamental signal voltage on which the pulse mode is based.

Capacitive Time Constant

The capacitive time constant of the previous circuit is \( \tau = RC \)

Two extreme cases are used in pulse mode.

Case 1: Small \( RC \) (\( \tau \ll t_c \))

Here the current flowing through the resistance is essentially equal to the instantaneous value of the current flowing in the detector. This configuration is used when high event rates or timing information are more important than accurate energy information.

Case 2: Large \( RC \) (\( \tau \gg t_c \))

Very little current flows in \( R \) during the charge collection time and the detector current is momentarily integrated on the capacitance. The voltage here has a long tail that is created as the capacitor discharges. If the time between pulses is sufficiently large, the capacitance will discharge through the resistance, returning the voltage across \( R \) to zero. This configuration is the most commonly used.
\( \tau \gg I_C \)

The magnitude of the maximum voltage is equal to \( Q \) (the total charge) divided by \( C \).

Instead of integrating the signal (as we should do in the case “small \( RC \)”), we simply measure \( V_{\text{max}} \) of the pulse.

**Figure 3**

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**See Next Chapters**

III. Pulse Height Spectra

IV. Counting Curves and Plateaus

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**Detectors in Radiation Spectroscopy**

1. Response to monoenergetic source of radiation

Fig. 4 shows the so-called response-function which represents the differential pulse height distribution.

**Figure 4**
Detectors in Radiation Spectroscopy

In both cases, good resolution and poor resolution, the same energy is deposited in the detector. This means that the area under both curves is the same. Although they are both centered on the same value, the difference comes from the fact that the detector with poor resolution has recorded a large amount of fluctuation from pulse to pulse. On the other extreme side, if these fluctuations are made smaller, the resolution becomes better and in an ideal case we should reach a clear spike or even an almost delta function. (There is still the broadening resulting from the nature of the measured energy – Uncertainty principle)

\[
R = \frac{\text{FWHM}}{H_0}
\]

The energy resolution \( R \) is defined by:

\[
R = \frac{\text{FWHM}}{H_0}
\]

unitless

where FWHM is the full width at half maximum and \( H_0 \) is the "centroid" of the gaussian signal.

\( R \) expresses the ability of the detector to distinguish between two radiations whose energies are near each other.

Roughly speaking the detector should be able to resolve two energies that are separated by more than one value of the detector FWHM.

Sources of Signal Noise

There are a number of potential sources of fluctuation in the response of a given detector:

1) Drift of the operating characteristics of the detector during the measurement.

2) Random noise within the detector and the whole instrumentation system (preamplifiers, amplifiers and the measurement electronics).

3) The statistical fluctuation (noise) which will be present whatever the remainder of the system is.
**Statistical Noise**

This is by far the most important noise to the detection signal. Statistical noise arises from the fact that the charge $Q$ generated within the detector is not a continuous variable but instead represents a discrete number of charge carriers. In an ion chamber the carriers are the ion pairs produced by the passage of the radiation, whereas in a scintillator the charge carriers are the number of electrons collected from the photocathode of the photomultiplier tube. In all cases this number is discrete and subject to random fluctuation from event to event even though exactly the same amount of energy is deposited in the detector.

**Estimate of Statistical Noise**

We can consider that the formation of each charge carrier is a Poisson process, i.e. the number of carriers created per event is small. If the total number $N$ of charge carriers is generated on the average, the standard deviation and thus the inherent statistical fluctuation is given by $\sqrt{N}$.

If this were the only source of fluctuation in the signal, the response function is a Gaussian, $N$ being a sufficiently large number (See Fig. 5).

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**FWHM**

The Gaussian function (Average $H_0$ and standard deviation $\sigma$) is:

$$G(\mu) = \frac{A}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(\mu-H_0)^2}{2\sigma^2}\right) = C \exp\left(-\frac{\mu^2}{2\sigma^2}\right)$$

The Full Width at Half Maximum is by definition:

$$\ln \left(\frac{1}{2}\right) = \left(\frac{h^2}{2\sigma^2}\right) \Rightarrow h^2 = 2\sigma^2 \ln 0.5 = 2\ln 2 \sigma$$

$$FWHM = 2h = 2\sqrt{2\ln 2} \sigma = 2.355 \sigma$$

**Resolution (Poisson Limit)**

For the majority of detectors, the average pulse height $H_0$ is proportional to $N$, i.e. $H_0 = K N$

The standard deviation $\sigma$ of the peak in the pulse height spectrum is $K \sqrt{N}$ and the FWHM is $2.355 K \sqrt{N}$.

A limiting resolution $R$ due to only to statistical fluctuation in the number of charge carriers is thus given by:

$$R_{\text{Poisson limit}} = \frac{FWHM}{H_0} = \frac{2.355 K \sqrt{N}}{K N} = 2.355 \sqrt{N}$$

Note the inverse proportionality between $R$ and $N$. $N$ should be greater than 55900 in order to achieve a resolution better than 1% ($R = 0.01$).
Fano Factor

Practically, measurements of $R$ give lower energy resolution by a factor of 3 or 4. This is essentially due to the fact that the processes that give rise to the formation of each charge carrier are not independent and in this case Poisson statistics is no more valid! The Fano factor is introduced in order to quantify the departure of (difference between) the observed statistical fluctuation in the number of charge carriers from pure Poisson statistics. It is defined by:

$$F = \frac{\text{observed variance in } N}{\text{Poisson Predicted variance (}} = N)$$

(13)

Overall FWHM

For typical semiconductor diode detectors the Fano factor is less than unity, while for scintillators this factor is almost unity.

In general one should add all "noises" and the overall FWHM in this case is defined by:

$$[FWHM]_{\text{overall}} = [FWHM]_{\text{statistical}} + [FWHM]_{\text{detector}} + [FWHM]_{\text{drift noise}}$$

(14)

VI. Detection Efficiency

What is Detection Efficiency?

Upon entry in the active volume, a charged particle, alpha or beta, and after traveling a small fraction of its range, enough pairs of ion-electron are created. Their number is sufficient to ensure that the resulting pulse is large enough to be recorded. In this case a detector will see every alpha or beta particle that enters its active volume. The detection efficiency, under these conditions, is 100%.

Things are different in the case of uncharged radiation since they must first undergo a significant interaction before being detected. The detection efficiencies in this case are often less than 100%. 
Absolute and Intrinsic efficiencies

We define the absolute efficiency by:

\[
\text{\( \epsilon_{\text{abs}} = \frac{\text{number of pulses recorded}}{\text{number of radiation quanta emitted by source}} \)}
\]

It is dependent on the detector nature and on the counting geometry (essentially the distance between the source of radiation and the detector).

The intrinsic efficiency defined by:

\[
\text{\( \epsilon_{\text{int}} = \frac{\text{number of pulses recorded}}{\text{number of radiation quanta incident on detector}} \)}
\]

no longer includes the solid angle subtended by the detector as an implicit factor.

Intrinsic Efficiency

The intrinsic efficiency depends on three factors:
1) The detector material,
2) Radiation energy,
3) Thickness of the detector in the direction of the incident radiation.

The dependence on the distance source-detector is still there, since it affects the range of the radiation in the detector.

Total Efficiency

If we accept all pulses from the detector, in this case one talks about total efficiency.

\[
\text{\( \epsilon_{\text{abs}} = \frac{\Omega}{4\pi} \)}
\]

(17)

In practice, some constraint is imposed on the output signal, using a discriminator or a gate, in order to eliminate very small pulses from electronic noise sources.

A threshold is imposed and the total efficiency is theoretically approached if this threshold is as small as possible.

Fig. 6 shows a hypothetical differential pulse height distribution.

Peak to Total ratio

The peak efficiency assumes that only those interactions that deposit the full-energy of the incident radiation are counted.

Figure 6
The number of events having deposited all their energy (full energy events) is simply obtained by integrating the area under the peak!

Events which deposited only a part of their energy are spread to the left of the peak.

A peak to total ratio relates the peak and the total efficiencies:

\[ \frac{\epsilon_{\text{peak}}}{\epsilon_{\text{total}}} \]

The peak to ratio efficiency is sometimes tabulated separately.

But it is often better to use the peak efficiency because the number of full energy events is not sensitive to perturbing effects such as scattering from surrounding objects or spurious noise.

Therefore, the peak efficiencies are compiled and tabulated and applied to a wide variety of laboratory conditions.

What is the "Dead Time"?

There will always be a minimum amount of time that must separate two events in order that they be recorded as two separate pulses.

This limiting time called the dead time of the counting system may be due to one of the following reasons (or both):

- Processes in the detector itself and/or
- Associated Electronics used for pulse processing and recording.
A. Models for Dead Time Behavior

There are two common models:

- Paralyzable response
- Nonparalyzable response

A detector is said to be paralyzable if it is affected by the radiation even if the signal is not processed. This could happen if the detector or its electronics are slow. Here pulses that arrive while the first pulse is being processed further delay the recovery of the counting system.

A detector is nonparalyzable if it has a fixed dead time. Here the pulses arriving while the first pulse is being processed are simply ignored.

These models represent idealized behavior, one or the other of which often adequately resembles the response of a real counting system.

Fig. 7 illustrates the fundamental assumptions of these models.

Definitions

Example – Extreme Cases

Six randomly spaced events in time in the detector:

- A paralyzable detector would record 3 events.
- A nonparalyzable one would record 4 events.

Real Case

A fixed time \( \tau \) is assumed to follow each true event that occurs during the “live period” of the detector. The true events that occur during the dead period are lost and assumed to have no effect whatsoever on the behavior of the detector.

In the example shown, the nonparalyzable detector would record four of the six events. In the case of the paralyzable detector, assuming the same \( \tau \), true events although not recorded by the detector extend the dead time by another period \( \tau \) following the lost event! Here only three events out of the true six ones are recorded.

This is an idealized case and the two models predict the same first-order losses and differ only when true events rates are high.

A practical case would be something between these two extremes.
Dead Time – Nonparalyzable Model

Let’s consider a steady-state source of radiation.

Let

\[ n = \text{true interaction rate} \]
\[ m = \text{recorded count rate} \]
\[ \tau = \text{system dead time} \]

We need to know the dead time in order to make appropriate corrections to measured data in order to have the true interaction rate \( n \).

In the nonparalyzable case (fixed \( \tau \)) the fraction of all time that the detector is dead is given simply by the product \( m \tau \). The total loss rate is here \( n m \tau \).

The rate of losses also equals \( n - m \) thus we have:

\[ n - m = n m \tau. \]

\[ \text{Nonparalyzable model} \quad (15) \]

Dead Time – Paralyzable Model

In the paralyzable case, as we said earlier, dead periods are not always of fixed length.

The rate \( m \) is identical to the rate of occurrences of time intervals between true events which exceed \( \tau \).

The distribution of intervals between random events occurring at an average rate \( n \) is given by:

\[ P_T(T) = n e^{-nT}dT \quad (19) \]

The probability of intervals larger than \( \tau \) can be obtained by integrating this distribution between \( \tau \) and \( \infty \), i.e.,

\[ \int_{\tau}^{\infty} n e^{-nT}dT = e^{-n\tau} \quad (20) \]

Fig. 8 shows the observed rate \( m \) vs. the true rate \( n \) for both models.

For low rates, the two modes give virtually the same result. For high rates it is another story!

The High Rate Case

In the paralyzable case, we cannot solve explicitly for \( n \). The solution is numerical (iteration)

\[ m = n e^{-n\tau}. \quad \text{Paralyzable model} \quad (21) \]

\[ \int p_T(t)dt = n e^{-n\tau}dT \quad (22) \]

Fig. 8 shows the observed rate \( m \) vs. the true rate \( n \) for both models.
B. Methods of Dead Time Measurement

More often the dead time will not be known or may vary with operating conditions and must therefore be measured directly.

The technique is to assume that one of the two modes is applicable and measure two different true rates which differ by a known ratio.

In the two-source method, the rate from the two sources separately is measured (values \( n_1 \) and \( n_2 \)) and a third measurement with the two sources combined is achieved (\( n_{12} \)).

Due to the fact that energy losses are not linear, the third measurement, \( n_{12} \), is smaller the sum \( n_1 + n_2 \). The discrepancy is used to compute the dead time.

The Two-Source Method

Let \( n_1 \), \( n_2 \) and \( n_{12} \) be the true counting rates (sample plus background) with sources 1, 2 and 1 and 2 combined respectively.

Let the corresponding measured rates be \( m_1 \), \( m_2 \) and \( m_{12} \).

Also let \( n_b \) and \( m_b \) be the true and measured background rates with both sources removed. Then we have:

\[
\frac{n_{12} - n_b}{n_{12} + n_b} = \left( \frac{n_1 - n_b}{n_1 + n_2} \right) + \left( \frac{n_2 - n_b}{n_1 + n_2} \right)
\]

\[
\Rightarrow n_{12} + n_b = n_1 + n_2
\]

Nonparalyzable Model

Assuming the nonparalyzable model, we have:

\[
\frac{m_{12}}{1-m_{12}\tau} + \frac{m_1}{1-m_1\tau} = \frac{m_1}{1-m_1\tau} + \frac{m_2}{1-m_2\tau}
\]

Solving this equation explicitly for \( \tau \) we get:

\[
\tau = \frac{X \left( 1 - \sqrt{1 - Z} \right)}{Y}
\]

where

\[
X = m_1 m_2 - m_b m_{12}
\]

\[
Y = m_1 m_2 \left( m_{12} + m_b \right) - m_1 m_{12} \left( m_1 + m_b \right)
\]

\[
Z = Y \left( m_1 + m_2 - m_{12} - m_b \right)
\]

\[
X^2
\]
No Background approximation

In the case where \( m_b \to 0 \), we have:

\[
X = m_1 m_2, \quad Y = m_1 m_2 m_{12}
\]

\[
Z = \frac{m_1 m_2 m_{12} (m_1 + m_2 - m_{12})}{(m_1 m_2)^2} = \frac{m_{12} (m_1 + m_2 - m_{12})}{m_1 m_2}
\]

and

\[
\tau = \frac{m_1 m_2 - [m_1 m_2 (m_{12} - m_1)(m_{12} - m_2)]^{1/2}}{m_1 m_2 m_{12}}
\] (26)

Approximations

Simplifications of Eq. 25 are based on many approximations. One should be aware that these approximations could introduce significant errors.

See Geiger-Müller Experiment for details on how to proceed.

The Short-lived isotope case:

See Knoll for the decaying source method for the calculations in the case of a short-lived isotope (pages 123-124) – Study Fig. 4.9 carefully.

Nonparalyzable mode:

\[
m e^{-\lambda t} = -n_i \tau m + n_i
\] (27-a)

Paralyzable mode:

\[
k_i \ln m = -n_i \tau e^{-\lambda t} + \ln n_i
\] (27-b)

C. Statistics of Dead Time Loses

Self Reading (pages 124-125)

D. Dead Time Loses from Pulsed Sources
Pulsed Sources

Until now we have assumed steady state sources. In some situations non steady state sources of radiation are encountered. For example:
- Electron linear accelerator used to generate high-energy X-rays can be operated to produce a few microsecond width with a repetition frequency of several kilohertz.

The analysis for dead time loses effects is different here from the one we detailed in the previous sections.

Analysis

The sketch below shows a source of pulsed source

We assume that the radiation intensity is constant throughout the duration $T$ of each pulse and that the pulses occur at a constant frequency $1/f$. Note that the time between two consecutive pulses is $1/f$.

Depending on the relative value of the detector dead time $\tau$, several conditions may apply:

$\tau$ vs. $T$

If $\tau$ is much smaller than $T$, the fact that the source is pulsed has little or negligible effect and this case becomes almost similar to the case of steady state sources.

If $\tau$ is less than $T$ but not by a large factor, only a small number of counts may be registered by the detector during a single pulse. This is a complicated situation which we will not treat here in this course.

If $\tau$ is larger than $T$ but less than the "off" time between pulses ($T \approx 1/f$), the following analysis is applied.

Counts

In the following we do not need to impose that the radiation be constant over the pulse length $T$ but we require that each radiation pulse be of the same intensity.

Using the same symbols in the steady state analysis, the probability of an observed count per second pulse is given by $m/f$.

The average number of true events per (source) pulse is by definition equal to $n/f$. 
Only one such count can be recorded, so the above expression is also the probability of recording a count per source pulse, i.e.

$$\tau > T$$

$$p(>0) = 1 - p(0) = 1 - e^{-\tau} = 1 - e^{-n/f}$$

(28)

Since the detector is live at the start of each pulse, a count will be recorded if at least one true event occurs during the pulse. Only one such count can be recorded, so the above expression is also the probability of recording a count per source pulse, i.e.

$$n = 1 - e^{-n/f}$$

Or

$$n = f \left( 1 - e^{-n/f} \right)$$

(29)

We see that the maximum observable counting rate is just the pulse repetition frequency.

No dependence on the length of the dead time or the detailed dead time behavior of the system (paralyzable or not).

Correction Formula

As usual we are interested in predicting the true rate ($n$)

Solving Eq. 29 for $n$ we get:

$$n = f \ln \left( 1 - \frac{m}{f} \right)$$

(30)

This condition is valid only under the conditions $T < \tau < \left( \frac{1}{f} - T \right)$.

In this case, the dead time loses are small under the condition $m \ll f$. At this limit we have:

$$n \rightarrow 0 \quad 1 - m/f$$

(31)

Effective Dead Time

Using the similarity with eq. 18, the previous relation can be viewed as predicting an effective dead time value of $\frac{1}{2} f$ in this low-loss limit.

Last remark: Since the value is now one half the source pulsing period, it can be many times larger than the actual physical dead time of the detection system!
Chapter 5
Gas-Filled Detectors